Chapter 5: Correlation

EXERCISE 5.1 [PAGE 63]

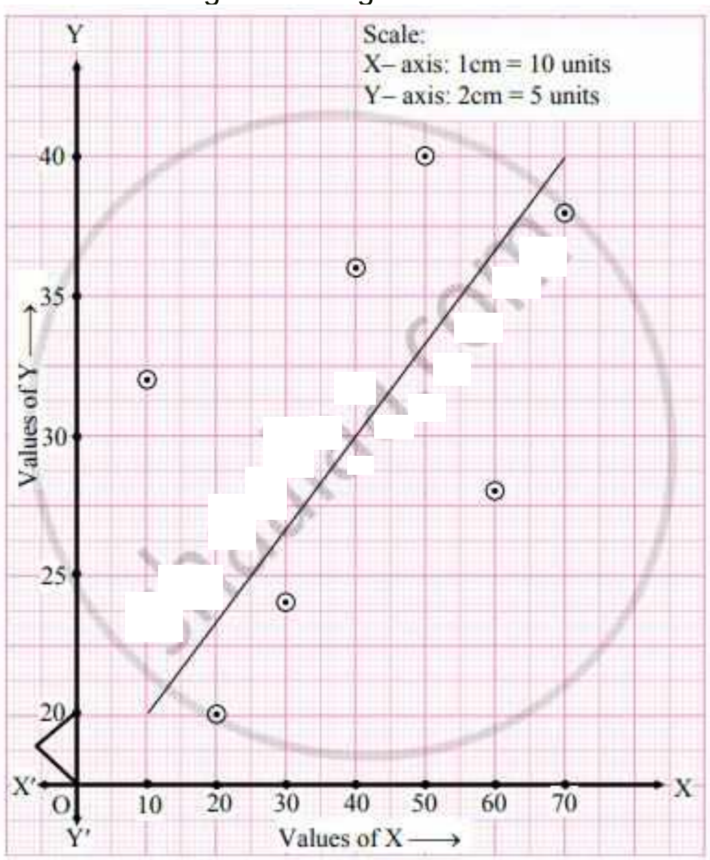
Exercise 5.1 | Q 1 | Page 63

Draw scatter diagram for the data given below and interpret it.

x	10	20	30	40	50	60	70
y	32	20	24	36	40	28	38

SOLUTION

The scatter diagram is as given below:



Since all the points lie in a band rising from left to right.
Therefore, there is a positive correlation between the values of X and Y respectively.

Exercise 5.1 | Q 2 | Page 63

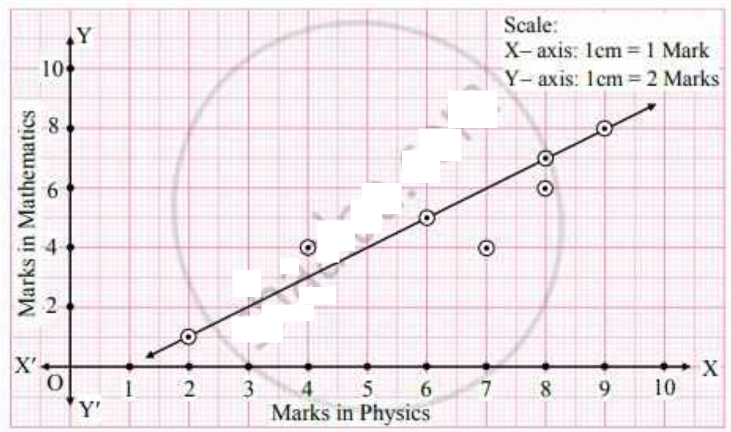
For the following data of marks of 7 students in physics (x) and Mathematics (y), draw scatter diagram and state the type of correlation.

x	8	6	2	4	7	8	9

y	6	5	1	4	4	7	8

SOLUTION

We take marks in Physics on X-axis and marks in Mathematics on Y-axis and plot the points as below.



We get a band of points rising from left to right. This indicates the positive correlation between marks in Physics and marks in Mathematics.

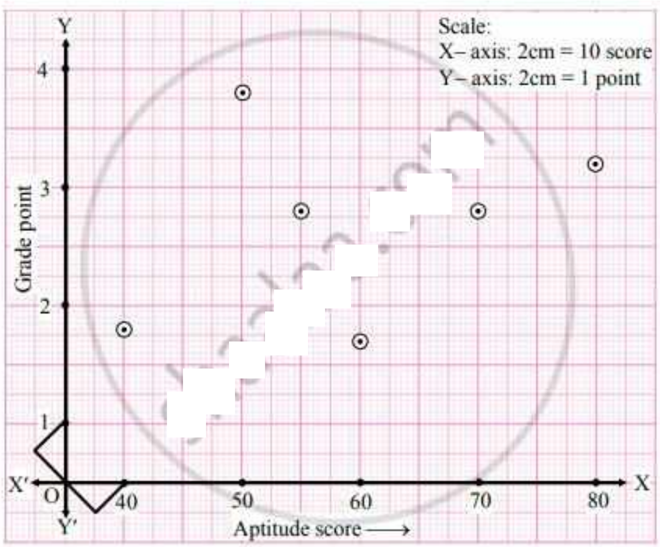
Exercise 5.1 | Q 3 | Page 63

Draw scatter diagram for the data given below. Is there any correlation between Aptitude score and Grade points?

Aptitude score	40	50	55	60	70	80
Grade points	1.8	3.5	2.8	1.7	2.8	3.2

SOLUTION

Aptitude score(x)	40	50	55	60	70	80
Grade points (y)	1.8	3.5	2.8	1.7	2.8	3.2



The points are completely scattered i.e., no trend is observed.

∴ There is no correlation between the Aptitude score (X) and Grade point (Y).

Exercise 5.1 | Q 4 | Page 63

Find correlation coefficient between x and y series for the following data.

n = 15, $\bar{x}$ = 25, $\bar{y}$ = 18, σ_x = 3.01, σ_y = 3.03, $\sum(x_i - \bar{x})(y_i - \bar{y}) = 122$

SOLUTION

here, n = 15, $\bar{x}$ = 25, $\bar{y}$ = 18, σ_x = 3.01, σ_y = 3.03, and $\sum(x_i - \bar{x})(y_i - \bar{y}) = 122$

Since, Cov (x, y) = $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$

∴ Cov (x, y) = $\frac{1}{15} \times 122$

= 8.13

Since, r = $\frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$

∴ = $\frac{8.13}{3.01 \times 3.03}$

= $\frac{8.13}{9.1203}$

∴ r = 0.89

Exercise 5.1 | Q 5 | Page 63

The correlation coefficient between two variables x and y are 0.48. The covariance is 36 and the variance of x is 16. Find the standard deviation of y.

SOLUTION

Given, r = 0.48, Cov (x, y) = 36

Since $\sigma_x^2 = 16$

$\therefore \sigma_x = 4$

Since, $r = \dfrac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$

$\therefore 0.48 = \dfrac{36}{4 \times \sigma_y}$

$\therefore \sigma_y = \dfrac{36}{4 \times 0.48} = \dfrac{9}{0.48}$

$= \dfrac{900}{48} = 18.75$

$\therefore$Standard deviation of y is 18.75.

Exercise 5.1 | Q 6. (i) | Page 63

In the following data one of the value of y is missing. Arithmetic means of x and y series are 6 and 8 respectively.

$$\left(\sqrt{2} = 1.4142\right)$$

x	6	2	10	4	8
y	9	11	?	8	7

Estimate missing observation

SOLUTION

Let the missing observation be 'a'

x	6	2	10	4	8
y	9	11	a	8	7

$\bar{x} = 6$, $\bar{y} = 8$, n = 5

$$\bar{y} = \frac{\sum_y}{n} = \frac{35 + a}{5}$$

$$\therefore 8 = \frac{35 + a}{5}$$

$\therefore$ a + 35 = 40

$\therefore$ a = 40 − 35

$\therefore$ a = 5

$\therefore$ Missing frequency = 5

Exercise 5.1 | Q 6. (ii) | Page 63

In the following data one of the value y of is missing. Arithmetic means of x and y series are 6 and 8 respectively.

$$\left(\sqrt{2} = 1.4142\right)$$

x	6	2	10	4	8
y	9	11	?	8	7

Calculate the correlation coefficient

SOLUTION

The missing observation be 'a'.

given, $\bar{x} = 6$, $\bar{y} = 8$, n = 5

$$\bar{y} = \frac{\sum y}{n}$$

$$\therefore 8 = \frac{35 + a}{5}$$

$\therefore$ 40 = 35 + a

$\therefore$ a = 5

We construct the following table:

x	y	x^2	y^2	xy
6	9	36	81	54
2	11	4	121	22
10	5	100	25	50
4	8	16	64	32
8	7	64	49	56
30	40	220	340	241

Here, $\sum_x = 30, \ \sum_y = 40$

$\sum x^2 = 220; \ \sum y^2 = 320$

$\sum xy = 214$ n = 5

∴ Karl Pearson's coefficient of correlation,

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$= \frac{5(214) - 30(40)}{\sqrt{5(220) - (30)^2}\sqrt{5(340) - (40)^2}}$$

$$= \frac{-130}{\sqrt{200}\sqrt{100}}$$

∴ = − 0.919

∴ There is high degree negative correlation.

Exercise 5.1 | Q 7 | Page 63

Find correlation coefficient from the following data, $\left[\text{Given} : \sqrt{3} = 1.732\right]$

x	3	6	2	9	5
y	4	5	8	6	7

SOLUTION

x	i	x_i^2	y_i^2	x_iy_i
3	4	9	16	12
6	5	36	25	30
2	8	4	64	16
9	6	81	36	54
5	7	25	49	35
25	**30**	**155**	**190**	**147**

From the table, we have

$n = 5, \sum x_i = 25, \sum y_i = 30, \sum x_i^2 = 155, \sum u_i^2 = 190, \sum x_iy_i = 147$

$$\bar{x} = \frac{\sum x_i}{n}$$

$$= \frac{25}{5}$$

$$= 5$$

$$\bar{y} = \frac{\sum y_i}{n}$$

$$= \frac{30}{5}$$

$$= 6$$

Since, $\mathrm{Cov}(x, y) = \frac{1}{n}\sum x_iy_i - \bar{x}\bar{y}$

$\therefore$ Cov (x, y) = $\frac{1}{5} \times 147 - (\times)$

= 29.4 – 30

= – 0.6

$\sigma_x^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

= $\frac{155}{5} - (5)^2$

= 31 – 25

$\therefore \sigma_x^2 = 6$

$\therefore \sigma_x = \sqrt{6}$

$\sigma_y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$

= $\frac{190}{5} - (6)^2$

= 38 – 36

$\therefore \sigma_y^2 = 2$

$\therefore \sigma_y = \sqrt{2}$

$\therefore \sigma_x\sigma_y = \sqrt{6}\sqrt{2} = \sqrt{12}$

= $2\sqrt{3}$

= 2(1.732) = 3.464

Thus, the correlation coefficient between x and y is

r = $\frac{\text{Cov}(x,y)}{\sigma_x \sigma y}$

$= \frac{-0.6}{3.464}$

$= -0.1732$

Exercise 5.1 | Q 8 | Page 63

Correlation coefficient between x and y is 0.3 and their covariance is 12. The variance of x is 9, Find the standard deviation of y.

SOLUTION

Given: r = 0.3, Cov (x, y) = 12,

$\sigma_x^2 = 9$

$\therefore \sigma_x = 3$

Karl Pearson's coefficient of correlation,

$r = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$

$\therefore 0.3 = \frac{12}{3 \times \sigma_y}$

$\therefore \sigma_y = \frac{12}{3 \times 0.3}$

$\therefore \sigma_y = \frac{4}{0.3}$

$\therefore \sigma_y = 13.33$

$\therefore$ The standard deviation of y is 13.33.

MISCELLANEOUS EXERCISE 5 [PAGES 63 - 64]

Miscellaneous Exercise 5 | Q 1 | Page 63

Two series of x and y with 50 items each have standard deviations of 4.8 and 3.5 respectively. If the sum of products of deviations of x and y series from respective arithmetic means is 420, then find the correlation coefficient between x and y.

SOLUTION

Given, n = 50, σ_x = 4.8, σ_y = 3.5, $\sum(x_i - \bar{x})(y_i - \bar{y})$ = 420

Cov (x, y) = $\frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})$

= $\frac{1}{50} \times 420$

∴ Cov (x, y) = 8.4

r = $\frac{Cov(x, y)}{\sigma_x \sigma_y}$

= $\frac{8.4}{(4.8)(3.5)}$

= $\frac{84 \times 10}{48 \times 35}$

= $\frac{1}{2}$

= 0.5

Miscellaneous Exercise 5 | Q 2 | Page 63

Find the number of pairs of observations from the following data,

r = 0.15, σ_y = 4, $\sum(x_i - \bar{x})(y_i - \bar{y})$ = 12, $\sum(x_i - \bar{x})^2$ = 40.

SOLUTION

Given, r = 0.15, σ_y = 4, $\sum(x_i - \bar{x})(y_i - \bar{y})$ = 12, $\sum(x_i - \bar{x})^2$ = 40

Since, $\sigma_x = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2} = \sqrt{\frac{40}{n}}$

Cov (x, y) = $\frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})$

= $\frac{1}{n} \times 12$

∴ Cov (x, y) = $\frac{12}{n}$

Since, r = $\frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$

∴ 0.15 = $\frac{\frac{12}{n}}{\sqrt{\frac{40}{n}} \times 4}$

∴ 0.15 = $\frac{3}{n \times \sqrt{\frac{40}{n}}}$

∴ 0.15 = $\frac{1}{\sqrt{n} \times \sqrt{40}}$

Squaring on both the sides, we get

0.0025 = $\frac{1}{n \times 40}$

∴ n = $\frac{1}{0.0025 \times 40}$

= $\frac{10000}{25 \times 40}$

= $\frac{10000}{1000}$

∴ n = 10

Miscellaneous Exercise 5 | Q 3 | Page 64

Given that r = 0.4, σ_y = 3, $\sum(x_i - \bar{x})(y_i - \bar{y})$ = 108, $\sum(x_i - \bar{x})^2$ = 900. Find the number of pairs of observations.

SOLUTION 1

Given, r = 0.4, $\sigma_y = 3, \sum(x_i - \bar{x})(y_i - \bar{y}) = 108, \sum(x_i - \bar{x})^2 = 900$

Cov (X, Y) = $\frac{1}{n}\sum(x_i - \bar{x})(y_i - \bar{y})$

= $\frac{1}{n} \times 108$

$\therefore$ Cov (X, Y) = $\frac{108}{n}$

$\sigma_x = \sqrt{\frac{1}{n} \times \sum(x_i - \bar{x})^2}$

= $\sqrt{\frac{1}{n} \times 900}$

= $\sqrt{\frac{900}{n}} = \frac{30}{\sqrt{n}}$

Since, r = $\frac{\text{Cov (X, Y)}}{\sigma_x \sigma_y}$

$\therefore$ 0.4 = $\frac{\frac{108}{n}}{\frac{30}{\sqrt{n}} \times 3}$

$\therefore$ 0.4 = $\frac{108}{n} \times \frac{\sqrt{n}}{30 \times 3}$

$\therefore$ 0.4 = $\frac{12}{10\sqrt{n}}$

$\therefore$ $\sqrt{n} = \frac{12}{4}$ = 3

$\therefore$ n = 9.

SOLUTION 2

Given, r = 0.4, $\sigma_y = 3$, $\sum(x_i - \bar{x})(y_i - \bar{y}) = 108$, $\sum (x_i - \bar{x})^2 = 900$.

Cov (x, y) = $\frac{1}{n}\sum(x_i - \bar{x}(y_i - \bar{y})$

= $\frac{1}{n} \times 108$

$\therefore$ Cov (x, y) = $\frac{108}{n}$

$\sigma_x = \sqrt{\frac{1}{n} \times \sum (x_i - \bar{x})^2}$

= $\sqrt{\frac{1}{n} \times 900}$

= $\sqrt{\frac{900}{n}} = \frac{30}{\sqrt{n}}$

Since, r = $\frac{\text{Cov }(x, y)}{\sigma_x \sigma_y}$

$\therefore$ 0.4 = $\frac{\frac{108}{n}}{\frac{30}{\sqrt{n}} \times 3}$

$\therefore$ 0.4 = $\frac{108}{n} \times \frac{\sqrt{n}}{30 \times 3}$

$\therefore$ 0.4 = $\frac{12}{10\sqrt{n}}$

$\therefore \sqrt{n} = \frac{12}{4}$ = 3

$\therefore$ n = 9

Miscellaneous Exercise 5 | Q 4 | Page 64

Given the following information, $\sum x_i^2 = 90$, $\sum x_i y_i = 60$, r = 0.8, $\sigma_y = 2.5$, where x_i and y_i are the deviations from their respective means, find the number of items.

SOLUTION

Here, r = 0.8, $\sum x_i^2 = 90$, $\sum x_i y_i = 60$, $\sigma_y = 2.5$,

Here, x_i and y_i are the deviations from their respective means.

$\therefore$ If X_i, Y_i are elements of x and y series respectively, then $X_i - \bar{x} = x_i$ and $Y_i - \bar{y} = y_i$

$\therefore \sum x_i y_i = \sum (X_i - \bar{x})(Y_i - \bar{y}) = 60$, $\sum x_i^2 = \sum (X_i - \bar{x})^2 = 90$

Now, $\sigma_x^2 = \frac{\sum (X_i - \bar{x})^2}{n}$

$\therefore \sigma_x^2 = \frac{90}{n}$

$\therefore \sigma_x = \sqrt{\frac{90}{n}}$

Also, Cov (X, Y) = $\frac{1}{n} \sum (X_i - \bar{x})(Y_i - \bar{y})$

$\therefore$ Cov (X, Y) = $\frac{60}{n}$

r = $\frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y}$

$$\therefore 0.8 = \frac{\frac{60}{n}}{\sqrt{\frac{90}{n}} \times 2.5}$$

$$\therefore 0.8 \times 2.5 \times \sqrt{\frac{90}{n}} = \frac{60}{n}$$

$$\therefore 2 \times \frac{\sqrt{90}}{\sqrt{n}} = \frac{60}{n}$$

$$\therefore \frac{n}{\sqrt{n}} = \frac{60}{2 \times \sqrt{90}}$$

$$\therefore \frac{\sqrt{n} \times \sqrt{n}}{\sqrt{n}} = \frac{30}{\sqrt{90}} = \frac{\sqrt{30} \times \sqrt{30}}{\sqrt{3}\sqrt{30}}$$

$$\therefore \sqrt{n} = \sqrt{10}$$

$\therefore$ n = 10

Miscellaneous Exercise 5 | Q 5 | Page 64

A sample of 5 items is taken from the production of a firm. Length and weight of 5 items are given below. [Given : $\sqrt{0.8823}$ = 0.9393]

Length (cm)	3	4	6	7	10
Weight (gm.)	9	11	14	15	16

Calculate the correlation coefficient between length and weight and interpret the result.

SOLUTION

Let length = x_i (in cm), Weight = y_i (in gm)

	x_i	y_i	x_i^2	y_i^2	$x_i y_i$
	3	9	9	81	27
	4	11	16	121	44
	6	14	36	196	84
	7	15	49	225	105
	10	16	100	256	160
Total	**30**	**65**	**210**	**879**	**420**

From the table, we have

n = 5, $\sum x_i$ = 30, $\sum y_i$ = 65, $\sum x_i^2$ = 210, $\sum y_i^2$ = 879, $\sum x_i y_i$ = 420

$$\bar{x} = \frac{\sum x_i}{n} = \frac{30}{5} = 6$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{65}{5} = 13$$

Cov (X, Y) = $\frac{1}{n}\sum x_i y_i - \bar{x}\bar{y}$

$= \frac{1}{5} \times 420 - 6 \times 13$

= 84 – 78

= 6

$$\sigma_x^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$= \frac{210}{5} - (6)^2$

= 42 – 36

$\therefore \sigma_x^2 = 6$

$\therefore \sigma_x = \sqrt{6}$

$$\sigma_y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$$

$= \frac{879}{5} - (13)^2$

= 175.8 – 169

$\sigma_y^2 = 6.8$

$\therefore \sigma_y = \sqrt{6.8}$

Thus, the coefficient of correlation between X and Y is

$$r = \frac{Cov(X, Y)}{\sigma_x \sigma_y}$$

$$= \frac{6}{\sqrt{6}\sqrt{6.8}}$$

$$= \sqrt{\frac{6}{6.8}}$$

$$= \sqrt{\frac{60}{68}}$$

$$= \sqrt{\frac{15}{17}}$$

$$= \sqrt{0.8823}$$

$\therefore r = 0.9393 \approx 0.94$

∴ The value of r indicates high degree of positive correlation between length and weight of items.

Miscellaneous Exercise 5 | Q 6 | Page 64

Calculate the correlation coefficient from the following data, and interpret it.

X	1	3	5	7	9	11	13
Y	12	10	8	6	4	2	0

SOLUTION

	x_i	y_i	x_i^2	y_i^2	$x_i y_i$
	1	12	1	144	12
	3	10	9	100	30
	5	8	25	64	40
	7	6	49	36	42
	9	4	81	16	36

	11	2	121	4	22
	13	0	169	0	0
Total	**49**	**42**	**455**	**364**	**182**

From the table, we have

$n = 7, \sum x_i = 49, \sum y_i = 42, \sum x_i^2 = 455, \sum y_i^2 = 364, \sum x_i y_i^2 = 182$

$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{49}{7} = 7,$

$\bar{y} = \frac{\sum y_i}{n} = \frac{42}{7} = 6$

$\text{Cov}(X, Y) = \frac{1}{n}\sum x_i y_i - \bar{x}\bar{y}$

$= \frac{1}{7} \times 182 - (7 \times 6)$

$= 26 - 42$

$\therefore \text{Cov}(X, Y) = -16$

$\sigma_x^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$= \frac{455}{7} - (7)^2$

$= 65 - 49$

$\therefore \sigma_x^2 = 16$

$\therefore \sigma_x = 4$

$$\sigma_y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$$

$$= \frac{364}{7} - (6)^2$$

$= 52 - 36$

$\sigma_y^2 = 16$

$\therefore \sigma_y = 4$

Thus, the coefficient of correlation between X and Y is

$$r = \frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y}$$

$$= \frac{-16}{4 \times 4}$$

$= -1$

$\therefore$ The value of r indicates perfect negative correlation between x and y.

Miscellaneous Exercise 5 | Q 7 | Page 64

Calculate the correlation coefficient from the following data and interpret it.

x	9	7	6	8	9	6	7
y	19	17	16	18	19	16	17

SOLUTION

	x_i	y_i	x_i^2	y_i^2	x_iy_i
	9	19	81	361	171
	7	17	49	289	119
	6	16	36	256	96
	8	18	64	324	144
	9	19	81	361	171

	6	16	36	256	96
	7	17	49	289	119
Total	**52**	**122**	**396**	**2136**	**916**

From the table, we have

n = 7, $\sum x_i$ = 52, $\sum y_i$ = 122, $\sum x_i^2$ = 396, $\sum y_i^2$ = 2136, $\sum x_i y_i$ = 916

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{52}{7}$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{122}{7}$$

$$\therefore \bar{x}\bar{y} = \frac{52 \times 122}{49} = \frac{6344}{49}$$

$$\text{Cov}(X, Y) = \frac{1}{n}\sum x_i y_i - \bar{x}\bar{y}$$

$$= \frac{916}{7} - \frac{6344}{49}$$

$$= \frac{6412 - 6344}{49}$$

$$= \frac{68}{49}$$

$$\sigma_x^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{396}{7} - \left(\frac{52}{7}\right)^2$$

$$= \frac{8772 - 2704}{49}$$

$$= \frac{68}{49}$$

$$\sigma_y^2 = \frac{\sum y_i^2}{n} - (\bar{y})^2$$

$$= \frac{2136}{7} - \left(\frac{122}{7}\right)^2$$

$$= \frac{14952 - 14884}{49}$$

$$= \frac{68}{49}$$

$$\therefore \sigma_x \sigma_y = \sqrt{\sigma_x^2 \sigma_y^2}$$

$$= \sqrt{\frac{68}{49} \times \frac{68}{49}}$$

$$= \frac{68}{49}$$

$$r = \frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y}$$

$$= \frac{\left(\frac{68}{49}\right)}{\frac{68}{49}} = 1$$

$\therefore$ The value of r indicates perfect positive correlation between x and y.

Miscellaneous Exercise 5 | Q 8. (i) | Page 64

If the correlation coefficient between x and y is 0.8, what is the correlation coefficient between 2x and y

SOLUTION

Correlation coefficient remains unaffected by the change of origin and scale.

i.e., if $\mathbf{u_i} = \frac{\mathbf{x_i - a}}{\mathbf{h}}$ and $\mathbf{v_i} = \frac{\mathbf{y_i - b}}{\mathbf{k}}$, then Corr (u, v) = ± Corr (x, y), according to the same or opposite signs of h and k.

$u_i = \frac{2(x_i - 0)}{1}, v_i = \frac{y_i - 0}{1}$

Here, h = 1 and k = 1 are of the same signs.

∴ Corr (u, v) = Corr (x, y) = 0.8

Miscellaneous Exercise 5 | Q 8. (ii) | Page 64

If the correlation coefficient between x and y is 0.8, what is the correlation coefficient between X/2 and y

SOLUTION

Correlation coefficient remains unaffected by the change of origin and scale.

i.e., if $\mathbf{u_i} = \frac{\mathbf{x_i - a}}{\mathbf{h}}$ and $\mathbf{v_i} = \frac{\mathbf{y_i - b}}{\mathbf{k}}$, then Corr (u, v) = ± Corr (x, y), according to the same or opposite signs of h and k.

$u_i = \frac{(x_i - 0)}{2}, v_i = \frac{y_i - 0}{1}$

Here, h = 1 and k = 1 are of the same signs.

∴ Corr (u, v) = Corr (x, y) = 0.8

Miscellaneous Exercise 5 | Q 8. (iii) | Page 64

If the correlation coefficient between x and y is 0.8, what is the correlation coefficient between x and 3y

SOLUTION

Correlation coefficient remains unaffected by the change of origin and scale.

i.e., if $\mathbf{u_i} = \frac{\mathbf{x_i - a}}{\mathbf{h}}$ and $\mathbf{v_i} = \frac{\mathbf{y_i - b}}{\mathbf{k}}$, then Corr (u, v) = ± Corr (x, y), according to the same or opposite signs of h and k.

Corr (x, 3y) = Corr (x, y) = 0.8

Miscellaneous Exercise 5 | Q 8. (iv) | Page 64

If the correlation coefficient between x and y is 0.8, what is the correlation coefficient between x – 5 and y – 3

SOLUTION

Correlation coefficient remains unaffected by the change of origin and scale.

i.e., if $u_i = \frac{x_i - a}{h}$ and $v_i = \frac{y_i - b}{k}$, then Corr (u, v) = ± Corr (x, y), according to the same or opposite signs of h and k.

Corr (x – 5, y – 3) = Corr (x, y) = 0.8

Miscellaneous Exercise 5 | Q 8. (v) | Page 64

If the correlation coefficient between x and y is 0.8, what is the correlation coefficient between x + 7 and y + 9

SOLUTION

Correlation coefficient remains unaffected by the change of origin and scale.

i.e., if $u_i = \frac{x_i - a}{h}$ and $v_i = \frac{y_i - b}{k}$, then Corr (u, v) = ± Corr (x, y), according to the same or opposite signs of h and k.

Corr (x + 7, y + 9) = Corr (x, y) = 0.8

Miscellaneous Exercise 5 | Q 8. (vi) | Page 64

If the correlation coefficient between x and y is 0.8, what is the correlation coefficient between $\frac{x - 5}{7}$ and $\frac{y - 3}{8}$?

SOLUTION

Correlation coefficient remains unaffected by the change of origin and scale.

i.e., if $u_i = \frac{x_i - a}{h}$ and $v_i = \frac{y_i - b}{k}$, then Corr (u, v) = ± Corr (x, y), according to the same or opposite signs of h and k.

$\text{Corr}\left(\frac{x - 5}{7}, \frac{y - 3}{8}\right)$ = Corr (x, y) = 0.8

Miscellaneous Exercise 5 | Q 9 | Page 64

In the calculation of the correlation coefficient between the height and weight of a group of students of a college, one investigator took the measurements in inches and pounds while the other investigator took the measurements in cm. and kg. Will they get the same value of the correlation coefficient or different values? Justify your answer.

SOLUTION

Coefficient of correlation is a ratio of covariance and standard deviations.
Since covariance and standard deviations are independent of units of measurement.
∴ Coefficient of correlation is also independent of units of measurement.
∴ Values of the coefficient of correlation obtained by first and second investigators are the same.